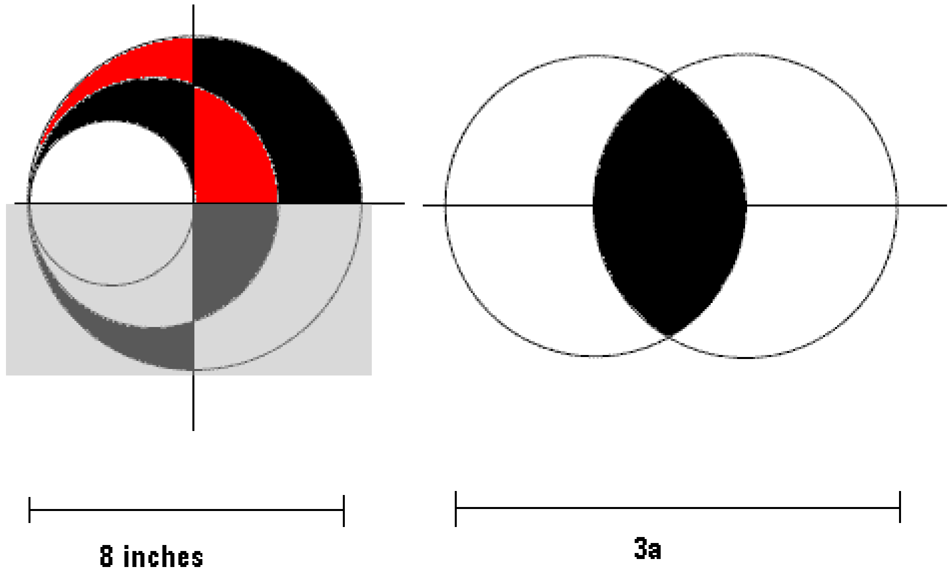
Solution (Circular Reasoning)



By symmetry, the shaded area in the figure on the left equals half of the area of the larger circle minus that of the smaller circle (see diagram above). This area is thus ½(16π – 4π) = 6π.

The circles on the right clearly have radius "a". We can introduce coordinates by taking the center of the circle on the left as the origin. The circles then have equations $x^2 + y^2 = a^2$ and $(x-a)^2 + y^2 = a^2$ and so they intersect at the two points on the line $x = \frac{1}{2}a$, that is, $(\frac{1}{2}a, \pm a\sqrt{3/4})$. Again, using symmetry, we can obtain this area by evaluating:

$$4\int_{\frac{a}{2}}^{a} \sqrt{a^2 - x^2}\, dx \;=\; a^2\left(\frac{4\pi - 3\sqrt{3}}{6}\right).$$

Setting this equal to 6π and taking the positive square root we obtain:

$$a \;=\; \sqrt{\frac{36\pi}{4\pi - 3\sqrt{3}}} \approx 3.9173 .$$