

Patterns and a Formula

Triangular numbers are so named because they can be arranged in a triangular configuration in a given pattern (see below). The first few triangular numbers are 1, 3, 6, 10, 15, 21, 28, 36, 45, ...

Consider the following pattern:

$$1 = 1$$

$$1 + 5 = 6$$

$$1 + 7 + 7 = 15$$

$$1 + 9 + 9 + 9 = 28$$

$$1 + 11 + 11 + 11 + 11 = 45$$

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This is a pattern that gives you the first few odd (the first, third, etc.) triangular numbers.

- 1) Generalize this pattern for all odd triangular numbers. That is, find an expression for the n^{th} term a_n , where n is odd.
- 2) Find a similar pattern for the even triangular numbers. (Hint: Consider using zero to begin) and generalize that pattern by writing the n^{th} term for the even triangular numbers.
- 3) Now give the formula for the n^{th} triangular number.