Student Engagement in a Quantitative Literacy Course

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Mathematics educators have been in the midst of reform for the past twenty years. Driven by a series of influential reports concerning the deteriorating state of mathematics education [including, *Everybody counts* (National Research Council, 1989) and *Moving beyond myths* (CMS2000, 1991)], many mathematics teachers have altered the way they teach: they appreciate the diversity of learning styles in a single classroom; they use computers and calculators in wise ways to extend, rather than replace, students' quantitative skills; and they understand the need to balance abstract topics with the practical uses of mathematics.

Most of these reforms have been directed largely at students in the "calculus pipeline," those taking precalculus or calculus courses. These so-called STEM (science, technology, engineering, and mathematics) students constitute a relatively small fraction of all college and university students who take mathematics courses. A large majority of all students who take mathematics courses are *not* calculus-bound and do so only to satisfy a core curriculum or general education requirement. These non-STEM students are primarily liberal arts students and they have not benefited greatly from the recent reforms. Furthermore, because of poor advising or lack of alternatives, many of these students mistakenly end up in the calculus pipeline, taking a college algebra course. The outcome is often catastrophic: students drop the course, fail the course, or leave the course with little appreciation of mathematics.

These observations point to the need for effective courses and programs designed specifically for non-technical students. Such initiatives appear under a variety of names including quantitative literacy (QL). The urgent need for effective QL courses and programs is expressed in recent reports commissioned by the MAA (Sons, 1995), AMATYC (Cohen, 1995), and the College Board (Steen, 1997). In 2001, the case for QL reappeared eloquently in a report that has inspired a new dialogue on the subject (Steen, 2001). In this article the challenges in designing and teaching QL courses are discussed. Courses taught at the University of Colorado at Denver that address these challenges are described. Results of a study of student engagement in this course are reported.

Challenges in Teaching QL Courses

Several observations help explain why teaching QL or liberal arts mathematics courses is challenging. These challenges arise from student attitudes and from a lack of consensus on the design of such courses.

Student Attitudes

Providing liberal arts students with a worthwhile experience in a QL course requires overcoming significant psychological obstacles. Students who take such courses often are "victims" of previous mathematics courses and instructors. As a result, they harbor genuine fears of mathematics, they have lost confidence in their quantitative skills, and they have little belief that mathematics might be of use in their future. A successful QL course cannot subject students to more of the same experiences they have had in previous mathematics courses.

The nature of students' anxiety and lack of confidence was revealed in a course questionnaire given to students at the beginning of the course. When asked what makes mathematics *difficult*, representative responses included:

- "Sometimes the verbiage—how what is being asked for is worded"
- "The lack of real-life examples"
- "I usually have trouble visualizing the grand scheme of a problem"
- "When it's explained in a way aimed at engineers and you're an artist"
- "Remembering formulas and procedures"

When asked what makes mathematics *frustrating*, selected responses were:

- "I have to work it all out by hand; the numbers don't align themselves in my head"
- "Knowing the way to solve it, then forgetting it five minutes later"

- "When I can't picture the problem in my head"
- "I feel like I think differently—as if I approach math in a different way; it seems like teachers always use the same approach to math and it's been one that I just don't follow; it makes me feel stupid."

Course Design

Given these attitudes, it is not surprising that students tend to be *disengaged* in such courses. It seems plausible that student engagement is a variable that influences students' success in the course. Thus, in designing and teaching a QL course it makes sense to focus on increasing student engagement.

Another challenge in teaching QL courses is that no current consensus about their content and expectations exists. Unlike a traditional algebra or calculus course, QL courses are quite varied in their content and purpose. Indeed, transferring such courses between institutions is often difficult.

The task of designing an effective QL course is further complicated by the role of algebra. The prevailing belief has been that a minimally educated person must be proficient with the abstractions and manipulations of algebra. For calculus-bound students, there is no question that algebra is a gatekeeper and its thorough mastery is essential. For a more general audience, such as liberal arts students, selected algebraic skills are important, but there are other, equally vital topics and skills.

The consequence of being selective about algebra and including a variety of practical topics is not a diluted mathematics course. The consequence is that mathematics becomes part of a larger set of skills that includes critical thinking, statistical reasoning, problem formulation, and written and oral communication. A QL approach allows students to see mathematics in a larger interdisciplinary setting that provides new problem-solving and decision-making powers. It presents mathematics *in context*, as a discipline that is connected to the world and essential to understanding that world. It provides students with a much broader survey of mathematics and statistics than afforded by other courses.

Meeting the Challenges: The CU-Denver Course

The above observations led to the development of a one-semester QL course at the University of Colorado at Denver. This course is taken by liberal arts students to satisfy a core curriculum requirement and is consistent with the overall principles expressed in recent QL reports. The course uses a text designed specifically for the course (Bennett, 2005) and teaching strategies that have evolved over several years. The course has three specific objectives that are intended to increase student engagement: **(a)** to strengthen and broaden students' quantitative skills; **(b)** to restore students' confidence in using those skills; and **(c)** to demonstrate the immediate relevance and applicability of mathematics to students' lives and careers.

Attitudes and Engagement

The class atmosphere is made informal, non-threatening, and supportive in the following ways.

- On the first day of class, the instructor informs the class that, "The first thing you need to know about me is that I want to be here." This statement conveys the *instructor's* engagement in the course, which may have an important modeling effect.
- Also on the first day of class, the instructor asks each person to discuss his/her most memorable mathematical moment. Not surprisingly, the majority of responses are negative. However, students feel that their past experience is "normalized" by hearing of others' similar experiences.
- Virtually all class periods begin with several minutes of *Number Patrol*, in which students (and the instructor) present personal or media-based stories that involve quantitative issues. Students are rewarded for making connections between their lives and the course. *Number Patrol* items include public opinion surveys, health studies, inflation figures, personal finance, and literary quotes.
- New material is introduced in short presentations, always leading with examples rather than abstract formulae. Students see that mathematics is only one step away from their lives, and that the goal of their study is to solve problems, not to learn mathematics for its own sake.
- Most classes include group problem solving sessions. Most students know only competitive mathematics classes in which they have always felt inferior to STEM students. In this class, the norm of mutual assistance and shared discovery is stated explicitly at the beginning of the course and actualized via group sessions.
- Students are encouraged to focus on the problem solving *process* rather than the final answer. It is not unusual to see students assemble an entire solution correctly and make a computational error on the last step; inevitably, the conclusion is that, "I'm no good at math!" Students in this class learn that much of mathematics is problem formulation—specifically translating words into mathematical ideas.
- Discussion and questions are always encouraged. Rather than train students to mimic patterns, the more effective approach is to listen carefully to how students think about problems and to guide them, *in their own language*, toward a solution.
- Memorization is not required. Rather, students should know which formula or method to use and how to use it.
- Effort is rewarded more than native mathematics ability. Roughly 70% of the final grade is determined by weekly assignments on which students may work in groups or seek help in office hours. Students complete a term project that counts for 15% of the final grade (topics range from the golden mean to analysis of lotteries, from mathematics of theater set design to college fund

investment strategies). Two open-book exams count for about 15% of the final grade.

Content

The topics presented in the course are chosen to prepare students for careers and lives that will be filled with quantitative information and decisions. A core of topics has emerged that can be supplemented by various secondary topics. The core of the course consists of the following topics, accompanied by a representative example:

1. Students must be equipped with strong **critical and logical thinking skills**, so they can navigate the media, make quantitative decisions, and be informed citizens.

   Example: The following ballot initiative appeared before Colorado voters in 1992: *Shall there be an amendment to the Colorado constitution to prohibit the state of Colorado and any of its political subdivisions from adopting or enforcing any law or policy which provides that homosexual, lesbian, or bisexual orientation, conduct, or relationships constitutes or entitles a person to claim any minority or protected status, quota preferences, or discrimination?*

   What does a *yes* vote mean?[1]

2. Students should have a strong **number sense** and be proficient at estimation, unit conversions, and the uses of percentages.

   Example: Suppose that the United States government decided to institute a national lottery, the proceeds of which would be used to retire the federal debt. Based on information that you gather about lottery finances, estimate how much money could be raised (after expenses and prizes) each week in a national lottery. How long would it take to pay off the federal debt assuming that the budget is exactly balanced every year in the future? Do you feel that this is a feasible strategy to pay off the federal debt?[2]

3. Students should be adept at **statistical reasoning** and be able to read a statistical study critically.

   Example: Suppose that 1000 people are given a drug test that is 98% accurate and that 50 of the people actually are drug users. What percentage

[1] A *yes* vote is a vote *against* protected status for homosexual, lesbian, or bisexual people.

[2] Assuming a federal debt of $6 trillion and lottery revenues of $50 million per week, it would take about 2300 years to pay off the debt.

of the positive tests are false positives (nonusers who test positive)? Discuss the implications of this calculation for the practice of drug testing.[3]

4. Students should possess the mathematical tools needed to make basic **financial decisions**.

   Example: Marie and Alex just paid $250,000 for a house. They made a down payment of $50,000 and assumed a 30-year $200,000 mortgage with a fixed annual interest rate of 7.50%. The house will serve as a residence for several years, but Marie and Alex also view it as an investment, as property values in the neighborhood are projected to increase at a rate of 5% per year in the near future. Suppose the couple sells the house after eight years. Neglecting income tax deductions, do they come out ahead on their investment?[4]

5. Students should understand the pervasive role that **probability** plays in our society, and be able to use that understanding to assess risks and make decisions.

   Example: Recent studies have shown that the probability of a woman getting a false report on a mammogram is 7 in 100 (or 0.07). What is the probability that a woman has at least one false report in ten annual mammograms?[5]

6. Students should understand **exponential growth** and know that it governs everything from populations and prices to tumors and drugs in the blood.

   Example: The world population is increasing at a rate of 1.3% per year. At this rate, how long will it take the population to double in size? At this rate, estimate the world population near the end of your lifetime.[6]

These examples illustrate typical problems in an effective QL course. They involve the application of relatively elementary mathematics to practical and often sophisticated situations. The problems are often open-ended, and serve to disabuse students of the belief that answers are always unique and given in the back of the book. Problems may involve the use of library or Internet resources for background information, and they have the goal of strengthening students' problem solving confidence and communication skills.

Over many semesters, these core topics have been supplemented by excursions into such issues as voting theory, apportionment, mathematics and the arts, graph

[3] About 28% of the positive tests are false positives.

[4] After 8 years, they have paid approximately $120,000 in interest and their property has appreciated by $119,360. So 8 years is very close to the break-even point.

[5] The probability is 0.52 or about 1 in 2.

[6] The doubling time is roughly 55 years. Assuming a current population of 6 billion people, a 20-year-old today would see a world population of roughly 13 billion 60 years from now.

Table 1: Twenty-seven Behaviors and Attitudes of Engagement

1. Raising my hand in class
2. Participating in small group discussions
3. Asking questions when I don't understand the instructor
4. Doing all the homework problems
5. Coming to class every day
6. Going to the professor's office hours to review or ask questions
7. Sitting toward the front of the class, where it's easier to pay attention
8. Thinking about the course between class meetings
9. Finding ways to make the course interesting to me
10. Taking good notes in class
11. Looking over class notes between classes
12. Really desiring to learn the material
13. Being confident that I can learn and do well in the class
14. Putting forth effort
15. Being determined to succeed
16. Being organized
17. Contacting the professor
18. Getting a good grade
19. Doing well on the tests
20. Staying up on the readings
21. Having fun in class
22. Helping fellow students
23. Making sure to study regularly
24. Finding ways to make the course material relevant to my life
25. Applying course material to my life
26. Figuring out what's expected of me in this class
27. Listening carefully in class

theory, energy and environmental problems. Often, students have a choice of supplementary topics, which may increase their motivation and involvement.

Evaluating the Course

Is this course meeting the objective of increasing engagement and changing attitudes? To assess the course, we administered beginning- and end-of-semester questionnaires. Students were asked a variety of questions that provided both qualitative and quantitative measures of attitudes and experiences with the course; approximately 35 students returned the questionnaires. We also assessed students' level of engagement by **(1)** asking students on a 5-point scale how engaged they were in the course, and **(2)** asking students to rate 27 attitudes and behaviors (see Table 1) on a scale from highly typical to highly atypical.

Our interest in student engagement arises from the plausible observation that engaged students are good learners and that effective teaching stimulates and sustains student engagement. Engagement (as opposed to mathematical background or proficiency) may be a good predictor of success in the course.

Table 2 presents the behaviors and attitudes that were most highly correlated with students' self-report of engagement at the beginning and end of the semester.

These lists are extremely informative for an instructor teaching a course about which students are admittedly less than eager. They suggest several strategies that might be used to increase students' receptivity and engagement; they are also borne out in the classroom. For example, fun and relevance are cited as important contributors to engagement and can be incorporated into the course with well-chosen examples and stories.

The change in behaviors between the beginning- and end-of-semester lists is also informative: *putting forth the effort* at the beginning of the semester appears to have been replaced by *desire to learn* at the end of the semester, and *listening in class* at the beginning of the semester appears to have been replaced by *thinking outside of class* at the end of the semester. Instructors might also be advised that *figuring out expectations* is a realistic student concern throughout the semester. Thus, it is very important to have a well designed syllabus and other guiding materials to give students clear information about expectations and grading.

The questionnaires also revealed changes of attitudes during the course of the semester. For example, female students reported a significant decrease in anxiety, a significant increase in confidence, and a significant increase in having fun in class. In addition, students who reported being unmotivated at the beginning of the semester reported higher levels of motivation at the end of the semester. Students also reported an increase in comfort and a benefit from working in groups. These changes may be attributable to students' low expectations and low proficiency at the start of the semester and to the realization that the QL course is a different course in which diligence and persistence lead to success.

Responses to open-ended questions also revealed changes in student attitudes. When asked how attitudes towards mathematics changed during the course, selected responses were:

- "I'm not as afraid as I used to be."
- "Math that deals with everyday things is important to learn."
- "This was a different math class; I'm used to solving math problems and not relating it to my life."
- "I will use math, such as topics taught in this class, in everyday life."
- "The utilitarian aspects of mathematics seem more clear and I have less anxiety about certain quantitative responsibilities."

Table 2: Correlations Between Engagement Behaviors and Self-reports of Engagement

Beginning of semester		**End of semester**	
Behavior	**Corr coeff**	**Behavior**	**Corr coeff**
Fun in class	0.733	Figuring out expectations	0.695
Figuring out expectations	0.666	Listening in class	0.689
Putting forth effort	0.579	Desire to learn	0.609
Listening in class	0.574	Thinking outside of class	0.524
Helping peers	0.531	Sitting near front of the class	0.504
Coming to class	0.529	Looking over notes	0.459
Making course relevant	0.525	Making course relevant	0.434
Applying course to my life	0.496	Fun in class	0.402

- "I realized that practice and reminders were what I needed, not a brain transplant."

Other responses to open-ended questions were informative and further illuminate what students may mean by having "fun in class." When asked what makes mathematics *stimulating*, selected responses were:

- "When I can see a problem solved in a certain way."
- "Figuring out a tough problem."
- "When I feel like I have enough of a handle to understand what I am doing and still feel challenged."
- "Learning problem solving skills seems to help in other areas of study."
- "Math is life; everything could/does involve math concepts."

When asked what makes mathematics fun, selected responses were:

- "Being able to help people."
- "Realizing that I get it—understand and can show/teach others."
- "Teaching others."
- "Low pressure exercises that take some thought."

These responses are useful when designing and teaching a QL course. For example, the fun that students have helping and teaching each other supports the use of group problem solving. The appreciation of the relevance of mathematics supports the continued use of examples from current news sources. The satisfaction that students find in successfully tackling problems suggests further work on problem solving skills.

The study also attempted to determine whether performance in the course was related to engagement. The relationship is difficult to establish because of the varied background of students. For example, students with a relatively strong mathematical background may excel with little engagement, while students with weak backgrounds (particularly older students) may also excel through diligence and hard work. However, the questionnaires did reveal the following relationships:

- The final course grade was most highly correlated with the following behaviors, as cited at the *beginning of the semester*: determination to succeed, getting a good grade, group discussion, and figuring out expectations.
- The final course grade was most highly correlated with the following behaviors, as cited at the *end of the semester*: getting a good grade, staying on top of the reading, and doing all of the homework.

The change in behaviors and attitudes between the beginning and end of the semester demonstrates the adaptivity of students as they seek strategies for success. Overall, it appears that performance is closely related to various behaviors that are easily associated with engagement: diligence, conscientious study, and of course, desire to get a good grade.

Future work will link these results on engagement with recent work in the psychology literature (Dweck, 2000) involving self-theories (incremental theory vs. entity theory) and student goals (performance goals vs. learning goals).

Conclusions

The authors have described the design, teaching, and assessment of a mathematics course for liberal arts students that conforms with current ideas about integrating QL into the undergraduate curriculum. The course can be taught effectively by creating a supportive and non-threatening atmosphere that heals student anxieties and strengthens student confidence. Equally important is the use of elementary skills and practical situations to demonstrate the relevance of mathematics to students' careers and lives. On the basis of the experience and study of the authors, it is believed that if student engagement is secured early in the course, it can change student attitudes favorably and lead to an effective learning experience.

The described efforts have been devoted to one particular QL course. Because such courses will have an increasingly vital role in the undergraduate curriculum, it is essential that mathematics educators understand the challenges involved with these courses and take the lead in designing and teaching effective courses. The ultimate goal is to move beyond individual courses and to weave QL throughout the curriculum. Although many practical issues remain to be resolved, this is a task in which mathematics departments must play a guiding role.

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