A PRECONDITIONER FOR GENERALIZED SADDLE POINT PROBLEMS

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Abstract

We consider the solution of systems of linear equations with the following block 2-by-2 structure:

$$\begin{bmatrix} A & B^T \\ B & -C \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{m\times n}$, $C \in \mathbb{R}^{m\times m}$, $f \in \mathbb{R}^n$, $g \in \mathbb{R}^m$, and $m \le n$. We further assume that matrices A, B and C are large and sparse. Systems of the form (1) arise in a variety of scientific and engineering applications, including computational fluid dynamics, mixed finite element approximation of elliptic partial differential equations, constrained optimization, etc.

We make the following assumptions:

- A has positive semidefinite symmetric part $H = \frac{1}{2}(A + A^T)$
- $Null(H) \cap Null(B) = \{0\}$
- $\mathrm{rank}(B) = m$
- C is symmetric positive semidefinite

which guarantee that (1) has exactly one solution. Our approach, described in detail in [3], is based on the symmetric/skew-symmetric iteration introduced in [1]. We prove convergence of the stationary iteration and we show how to optimize the rate of convergence as a function of method parameters. When GMRES acceleration is used, an optimal (h-independent) method is obtained for some model elliptic PDE problems in mixed form [2].

The performance of the preconditioner is illustrated with numerical experiments on various linear systems, including some arising in the solution of Stokes and Navier–Stokes problems. This is joint work with Martin Gander (McGill University) and Gene Golub (Stanford University).

References

[1] Z. Z. Bai, G. H. Golub, and M. K. Ng, *Hermitian and skew-Hermitian splitting methods for non-Hermitian positive definite linear systems*, SIAM J. Matrix Anal. Applic., to appear (2003).

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[3] M. Benzi and G. H. Golub, *An iterative method for generalized saddle point problems*, Technical Report SCCM-02-14, Scientific Computing and Computational Mathematics Program, Department of Computer Science, Stanford University, 2002 (Available online, see URL above).